to the Editor

Shape Factors in Multiple-pipe Systems

Richard C. Faulkner and Robert V. Andrews

Texas Agricultural and Mechanical College System, College Station, Texas

Problems in conductive heat transfer frequently do not lend themselves to simple mathematical treatment. Except for a few geometrical configurations such as parallel plates and concentric cylinders, an exact mathematical solution is often either impossible or difficult, owing to the irregular shape of the body conducting heat and the complicated boundary conditions involved. Through the use of experimentally determined shape factors, many of these problems can be solved easily.

A simple, inexpensive means for the experimental determination of shape factors has been presented by Andrews(*1*). The method consists essentially of drawing with a silver paint on a conducting paper the figure whose shape factor is desired. The electrical resistance of this figure is then compared to the electrical resistance of a standard figure whose shape factor can be calculated mathematically. Equation (1) is then used to calculate the shape factor of the desired figure. The shape

$$\text{shape factor}_{\text{unknown}} = SF_{\text{std.}} \; R_{\text{std.}}/R_{\text{unknown}} \qquad (1)$$

where
SF = shape factor
R = resistance in ohms

factor so determined is then used in Equation (2) to calculate the heat transferred by conduction. Figure 1 shows the electrical circuit used to obtain accurate values of the resistances which are used in Equation (1).

$$q = -k\,(SF)\,(\Delta T) \qquad (2)$$

where
k = thermal conductivity, B.t.u./(hr.) (sq.ft.) (°F./ft.)
q = B.t.u./hr.
ΔT = thermal driving force, °F.

The shape factors obtained by this method are independent of the thermal conductivity of the material transferring heat or of the terminal temperatures of the heat flow path. As the units of the shape factor are length, it may be considered to be an effective area for heat transfer divided by a unit effective length of heat transfer path.

The use of buried conduits containing more than one pipe is quite common; i.e., steam tracing in connection with the transportation of substances which may solidify or become viscous if allowed to fall below a certain temperature. Shape factors for such pipe configurations are shown in Figures 2, 3 and 4. In these figures all dimensions are shown as a fraction of D_o, the diameter of the outer conduit containing the multiple pipes. Because of this, the shape factors in the

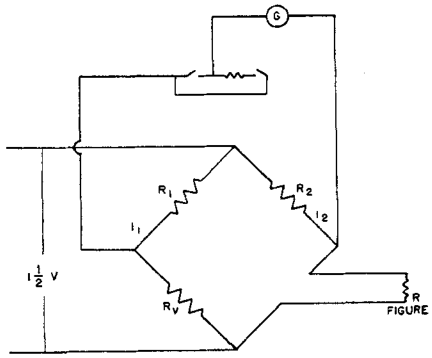

Fig. 1. Effective circuit diagram.

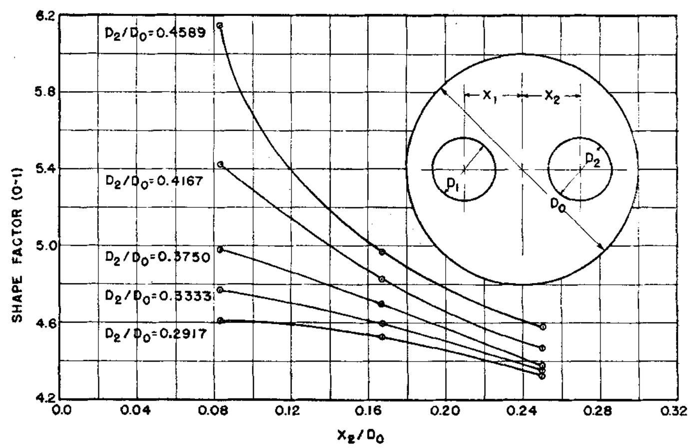

Fig. 2. Shape factor (0-1) vs. x_2/D_0 at $x_1/D_0 = 0.2500$ and $D_1/D_0 = 0.1667$.

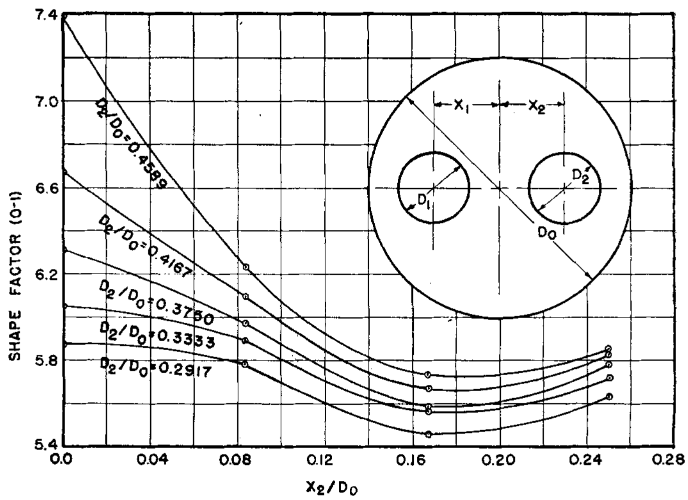

Fig. 3. Shape factor (0 - 1) vs. x_2/D_0 at $x_1/D_0 = 0.3333$ and $D_1/D_0 = 0.1667$.

figures are not limited to one set of dimensions, but may be applied to any geometry having the same diameter ratio. For example, if $X_2 = 3$ in. and $D_o = 12$ in. or if $X_2 = 1.5$ in. and $D_o = 6$ in., the same ratio of $X_2/D_o = 0.25$ is obtained, and the same shape factor applies. The designation of the pipes inside the conduit as pipe 1 and pipe 2 has no significance except to aid in the use of the curves.

Figures 2 and 3 give values of the shape factor between the outer conduit and either of the two pipes contained in the conduit for certain specific dimension ratios as parameters. Figure 4 gives values of the shape factor between the two pipes contained in the conduit for certain specific dimension ratios as parameters. As a generalization it was found that the shape factor for conductive heat transfer between either of the two pipes and the outer conduit could be approximated by Equation (3) to give an answer accurate within 3% of the correct value.

$$SF_{(0-1) \text{ or } (0-2)} = \frac{6.472}{\cosh^{-1}\left[\dfrac{a_1^{\,2}+a_2^{\,2}-d^2}{2a_1\, a_2}\right]} \qquad (3)$$

Equation (3) is actually the equation for the shape factor of heat transfer between cylinders, one inside the other but not concentric. Apparently the presence of the second pipe inside the conduit has little effect on the shape factor for heat transfer between the conduit and the other pipe. Shape factors for heat transfer between the two pipes contained in the conduit did not follow an equation for heat transfer between adjacent cylinders, and no other general correlation for this case was found. Instead, graphical determination of shape factors from Figure 4 or similar curves proved the best method of determination.

For conduits containing three pipes, no general correlation for shape factors was found. Figures 5 and 6 present shape-factor data for a three-pipe conduit of common commercial design. In these figures parameters of $D_1/D_o = 0.5$ and $D_2/D_o = 0.2917$, 0.3333, and 0.3750 are used. In the ranges studied, the shape factors $(0-3)$ and $(1-3)$ are independent of the variation of D_2/D_o. It is also of interest to note that the shape of $(0-3)$ was always larger than that of $(1-3)$ or $(2-3)$. This would indicate that if pipe 3 were to be used for heating, the distance X_3 should be made as small as possible to lower the heat transfer from pipe 3 to pipe 0, and so increase the heat transfer from pipes 3 to 1 and 3 to 2; that is, the heating pipe should be as close to the center of the enclosing conduit as possible rather than merely being located as close as possible to the pipes to be heated.

The two-dimensional method of determining shape factors for multiple-pipe systems which is described here provides a rapid method of solving many of the problems commonly encountered in industry. The method is extremely simple and inexpensive and requires a minimum of equipment and skill.

Literature Cited

1. Andrews, R. V., *Chem. Eng. Progr.*, 51, 67F (1955).

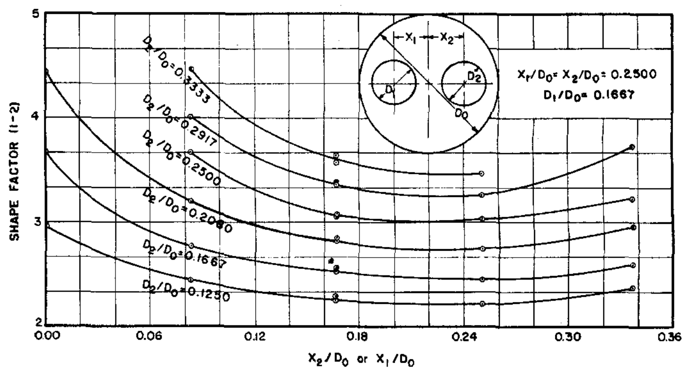

Fig. 4. Shape factor (1-2) vs. x/D_0.

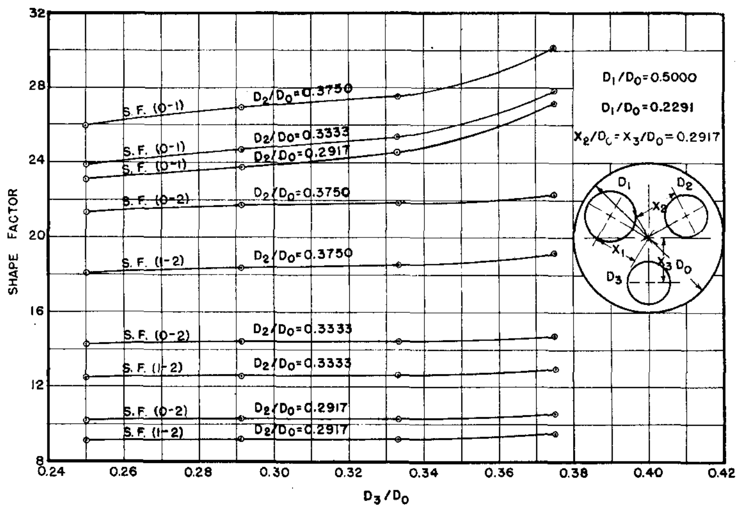

Fig. 5. Shape factor for three-pipe conduit.

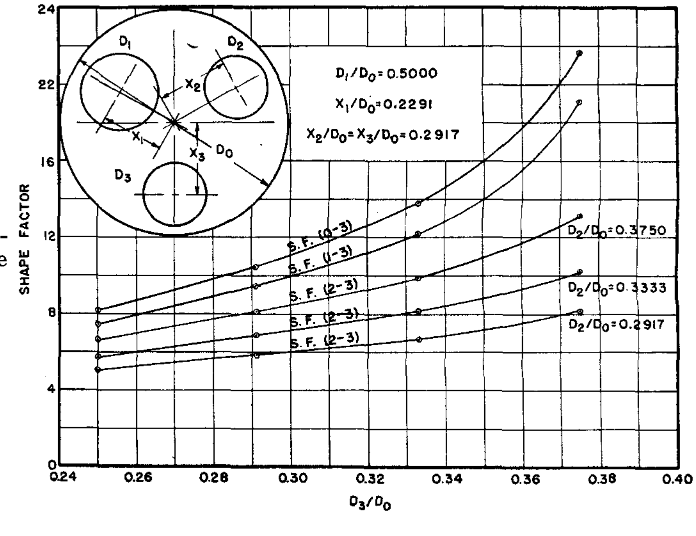

Fig. 6. Shape factor for three-pipe conduit.